

CALCULUS DERIVATIVES AND LIMITS

DERIVATIVE DEFINITION

$$\frac{d}{dx}(f(x)) = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

BASIC PROPERTIES

$$\begin{aligned}(cf(x))' &= c(f'(x)) \\ (f(x) \pm g(x))' &= f'(x) \pm g'(x) \\ \frac{d}{dx}(c) &= 0\end{aligned}$$

MEAN VALUE THEOREM

If f is differentiable on the interval (a, b) and continuous at the end points there exists a c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

PRODUCT RULE

$$(f(x)g(x))' = f(x)'g(x) + f(x)g(x)'$$

QUOTIENT RULE

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

POWER RULE

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

CHAIN RULE

$$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$$

COMMON DERIVATIVES

$$\begin{aligned}\frac{d}{dx}(x) &= 1 \\ \frac{d}{dx}(\sin x) &= \cos x \\ \frac{d}{dx}(\cos x) &= -\sin x \\ \frac{d}{dx}(\tan x) &= \sec^2 x \\ \frac{d}{dx}(\sec x) &= \sec x \tan x \\ \frac{d}{dx}(\csc x) &= -\csc x \cot x \\ \frac{d}{dx}(\cot x) &= -\csc^2 x \\ \frac{d}{dx}(\sin^{-1} x) &= \frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx}(\cos^{-1} x) &= -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx}(\tan^{-1} x) &= \frac{1}{1+x^2} \\ \frac{d}{dx}(a^x) &= a^x \ln(a) \\ \frac{d}{dx}(e^x) &= e^x \\ \frac{d}{dx}(\ln(x)) &= \frac{1}{x}, x > 0 \\ \frac{d}{dx}(\ln|x|) &= \frac{1}{x} \\ \frac{d}{dx}(\log_a(x)) &= \frac{1}{x \ln(a)}\end{aligned}$$

CHAIN RULE AND OTHER EXAMPLES

$$\begin{aligned}\frac{d}{dx}([f(x)]^n) &= n[f(x)]^{n-1}f'(x) \\ \frac{d}{dx}(e^{f(x)}) &= f'(x)e^{f(x)} \\ \frac{d}{dx}(\ln[f(x)]) &= \frac{f'(x)}{f(x)} \\ \frac{d}{dx}(\sin[f(x)]) &= f'(x)\cos[f(x)] \\ \frac{d}{dx}(\cos[f(x)]) &= -f'(x)\sin[f(x)] \\ \frac{d}{dx}(\tan[f(x)]) &= f'(x)\sec^2[f(x)] \\ \frac{d}{dx}(\sec[f(x)]) &= f'(x)\sec[f(x)]\tan[f(x)] \\ \frac{d}{dx}(\tan^{-1}[f(x)]) &= \frac{f'(x)}{1+[f(x)]^2} \\ \frac{d}{dx}(f(x)^{g(x)}) &= f(x)^{g(x)}\left(\frac{g(x)f'(x)}{f(x)} + \ln(f(x))g'(x)\right)\end{aligned}$$

PROPERTIES OF LIMITS

These properties require that the limit of $f(x)$ and $g(x)$ exist

$$\begin{aligned}\lim_{x \rightarrow a}[cf(x)] &= c \lim_{x \rightarrow a} f(x) \\ \lim_{x \rightarrow a}[f(x) \pm g(x)] &= \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) \\ \lim_{x \rightarrow a}[f(x)g(x)] &= \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x) \\ \lim_{x \rightarrow a}\left[\frac{f(x)}{g(x)}\right] &= \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \text{ if } \lim_{x \rightarrow a} g(x) \neq 0 \\ \lim_{x \rightarrow a}[f(x)]^n &= \left[\lim_{x \rightarrow a} f(x)\right]^n\end{aligned}$$

LIMIT EVALUATION METHOD – FACTOR AND CANCEL

$$\lim_{x \rightarrow -3} \frac{x^2 - x - 12}{x^2 + 3x} = \lim_{x \rightarrow -3} \frac{(x+3)(x-4)}{x(x+3)} = \lim_{x \rightarrow -3} \frac{(x-4)}{x} = \frac{7}{3}$$

L'HOPITAL'S RULE

$$\text{If } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\pm\infty}{\pm\infty} \text{ then } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

LIMIT EVALUATIONS AT $+\infty$

$$\begin{aligned}\lim_{x \rightarrow \infty} e^x &= \infty \text{ and } \lim_{x \rightarrow -\infty} e^x = 0 \\ \lim_{x \rightarrow \infty} \ln(x) &= \infty \text{ and } \lim_{x \rightarrow 0^+} \ln(x) = -\infty \\ \text{If } r > 0 \text{ then } \lim_{x \rightarrow \infty} \frac{c}{x^r} &= 0 \\ \text{If } r > 0 \text{ \& } x^r \text{ is real for } x < 0 \text{ then } \lim_{x \rightarrow -\infty} \frac{c}{x^r} &= 0 \\ \lim_{x \rightarrow \pm\infty} x^r &= \infty \text{ for even } r \\ \lim_{x \rightarrow \infty} x^r = \infty \text{ \& } \lim_{x \rightarrow -\infty} x^r = -\infty &\text{ for odd } r\end{aligned}$$

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